

Multiple Scattering, Parton Energy Loss and Modified Fragmentation Functions in Deeply Inelastic eA Scattering

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Modified quark fragmentation functions in deeply inelastic eA collisions and their QCD evolution equations are derived for the first time in the framework of multiple parton scattering. Induced radiation gives rise to additional terms in the evolution equations and thus softens the modified quark fragmentation functions. The results in the next-leading-twist depend on both diagonal and off-diagonal twist-four parton distributions and the combination of which clearly manifests the LPM interference pattern. The predicted modification depends quadratically on the nuclear size ($A^{2/3}$). Generalization to the case of hot QCD medium is also discussed.

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The propagation of an energetic parton and its induced energy loss has been proposed as a probe of the properties of dense matter formed in high-energy nuclear collisions [1,2]. Recent theoretical studies [3–6] show that a fast parton will lose a significant amount of energy via induced radiation when it propagates through a hot partonic matter. One cannot directly measure the energy loss of partons because they are not final experimentally observed particles. However, parton energy loss does lead to modification of the final particle spectra. Therefore, one can only study the parton energy loss indirectly by measuring the modification of the parton fragmentation functions in semi-inclusive processes like eA or γ -jet events in AA collisions [7] or the inclusive spectra at large transverse momentum [2,8].

In this Letter, we report our first study and derivation of the QCD evolution equations for the medium-modified fragmentation functions in the simplest case of deeply inelastic eA scattering (DIS). The induced gluon radiation due to multiple parton scattering gives rise to additional terms in the modified QCD evolution equations that soften the modified fragmentation functions. Utilizing the generalized factorization of higher-twist (HT) parton distributions [9], we show that these additional HT terms depend on both the diagonal and off-diagonal twist-four parton distributions, the combination of which clearly manifests the Landau-Migdal-Pomeranchuk (LPM) interference pattern. Using estimates of these twist-four parton matrix elements from other processes such as the p_T broadening of Drell-Yan dilepton in pA collisions, we predict the modification of the effective quark fragmentation functions and their dependence on the parton energy and nuclear size. We also estimate the quark energy loss defined as the total energy carried by gluons from induced radiation.

We consider the following semi-inclusive process in the deeply inelastic eA scattering, $e(L_1) + A(p) \rightarrow e(L_2) + h(\ell_h) + X$, where L_1 and L_2 are the four momenta of the incoming and the outgoing leptons, ℓ_h is the ob-

served hadron momentum, p and $q = L_2 - L_1$ denoted as $p = [p^+, 0, \mathbf{0}_\perp]$, $q = [-Q^2/2q^-, q^-, \mathbf{0}_\perp]$, are the momentum per nucleon in the nucleus with the atomic number A and the momentum transfer, respectively. The differential cross section for the semi-inclusive process can be expressed as

$$E_{L_2} E_{\ell_h} \frac{d\sigma_{\text{DIS}}^h}{d^3 L_2 d^3 \ell_h} = \frac{\alpha_{\text{EM}}^2}{2\pi s} \frac{1}{Q^4} L_{\mu\nu} E_{\ell_h} \frac{dW^{\mu\nu}}{d^3 \ell_h} \quad (1)$$

where $s = (p + L_1)^2$ and α_{EM} is the electromagnetic (EM) coupling constant. The leptonic tensor is given by $L_{\mu\nu} = 1/2 \text{Tr}(\gamma \cdot L_1 \gamma_\mu \gamma \cdot L_2 \gamma_\nu)$ while the semi-inclusive hadronic tensor is defined as,

$$E_{\ell_h} \frac{dW_{\mu\nu}}{d^3 \ell_h} = \frac{1}{2} \sum_X \langle A | J_\mu(0) | X, h \rangle \langle X, h | J_\nu(0) | A \rangle \times 2\pi \delta^4(q + p - p_X - \ell_h) \quad (2)$$

where \sum_X runs over all possible final states and $J_\mu = \sum_q e_q \bar{\psi}_q \gamma_\mu \psi_q$ is the hadronic EM current.

In the parton model with collinear factorization approximation and to the leading-twist (LT) the semi-inclusive cross section factorizes into a product of parton distributions, parton fragmentation functions and the partonic cross section. Therefore, to the leading order in α_s ,

$$\begin{aligned} \frac{dW_{\mu\nu}^S}{dz_h} &= \sum_q e_q^2 \int dx f_q^A(x, \mu_I^2) H_{\mu\nu}^{(0)}(x, p, q) D_{q \rightarrow h}(z_h, \mu^2) \\ H_{\mu\nu}^{(0)}(x, p, q) &= \frac{1}{2} \text{Tr}(\gamma \cdot p \gamma_\mu \gamma \cdot (q + xp) \gamma_\nu) \frac{2\pi}{2p \cdot q} \delta(x - x_B), \end{aligned} \quad (3)$$

where the momentum fraction carried by the hadron is defined as $z_h = \ell_h^-/q^-$ and $x_B = Q^2/2p^+q^-$ is the Bjorken variable. μ_I^2 and μ^2 are the factorization scales for the initial quark distributions $f_q^A(x, \mu_I^2)$ in a nucleus and the fragmentation functions $D_{q \rightarrow h}(z_h, \mu^2)$, respectively. Including all leading log radiative corrections, the

renormalized quark fragmentation function $D_{q \rightarrow h}(z_h, \mu^2)$ satisfies the QCD evolution equation [10].

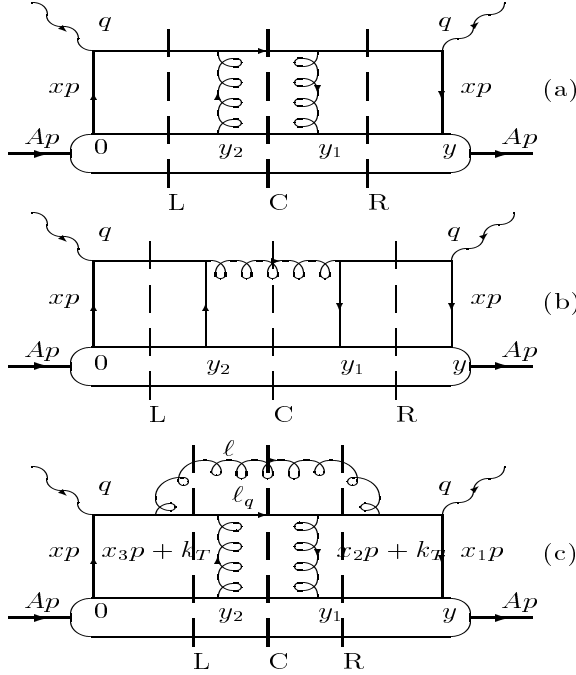


FIG. 1. Diagrams for rescattering with gluons (a) and quarks (b) without and with gluon radiation (c) in deeply inelastic eA scattering. Possible cuts are shown by the dashed lines

In this paper we will consider contributions of quark rescattering with partons from another nucleus inside the nucleus. Such contributions are proportional to the nuclear size $A^{1/3}$ [9]. For large enough A , we can neglect other A -independent HT effects. For large Q^2 in DIS, it may suffice to only consider one rescattering. The contributions of one rescattering can be treated as HT corrections to the LT results. We work in a framework [12] in which the twist-four contributions can be expressed as the convolution of the partonic hard parts and four-parton matrix elements [9]. At the lowest order, rescattering without gluon radiation as shown in Fig. 1(a) broadens the transverse momentum of the leading jet [11] but contribute little to parton energy loss. One can also neglect rescattering with another quark in Fig. 1(b).

The dominant HT contributions to the QCD evolution of the fragmentation functions come from radiative processes involving rescattering with a gluon from another nucleus as illustrated by the central-cut diagram in Fig. 1(c). Kinematics only allows two poles, one at each side of the central-cut, out of the four propagators in the diagram. This leads to four possible combinations each give different momentum fractions to the initial partons. In one case, the initial gluon has $x_2 = x_L + x_D$ which is finite when $k_T \rightarrow 0$, where

$$x_L = \frac{\ell_T^2}{2p^+q^-z(1-z)}; \quad x_D = \frac{k_T^2 - 2\vec{k}_T \cdot \vec{\ell}_T}{2p^+q^-z}, \quad (4)$$

ℓ_T is the transverse momentum of the radiated gluon, k_T is the initial gluon's intrinsic transverse momentum, and $z = \ell_q^-/q^-$ is the momentum fraction carried by the final quark. This corresponds to gluon radiation induced by the rescattering and is referred to as a double-hard process. In another combination, $x_2 = x_D$ which vanishes when $k_T \rightarrow 0$. In this case the rescattering is soft and the gluon radiation is induced by the initial hard photon-quark scattering. Such a process is called hard-soft. The four contributions from Fig. 1(c) correspond to these two distinct processes and their interferences. Their sum has the form,

$$H_{\mu\nu}^{D(1)} \propto (1 - e^{-ix_L p^+ y_2^-})(1 - e^{-ix_L p^+(y^- - y_1^-)}) \times e^{ix_D p^+(y_1^- - y_2^-)}. \quad (5)$$

This clearly manifests the LPM interference pattern caused by the destructive interferences between hard-soft and double-hard processes. The interference pattern is dictated by the gluon's formation time, $\tau_f \equiv 1/x_L p^+$, relative to the nuclear size. The two processes completely cancel each other in the collinear limit when $\ell_T \rightarrow 0$. Diagrams involving three-gluon vertices have exactly the same structure as Fig. 1(c), except that they have different momentum dependence and color factor in the hard part.

We have considered all together 23 possible cut diagrams, 14 of them are interferences between no and double rescattering (shown as the left and right-cut diagrams in Fig. 1(c)) which cancel some of the contributions from central-cut diagrams. Including virtual corrections, we obtain [13] the leading HT contribution from rescattering processes,

$$\begin{aligned} \frac{dW_{\mu\nu}^D}{dz_h} = & \sum_q e_q^2 \int dx H_{\mu\nu}^{(0)}(x, p, q) \frac{2\pi\alpha_s}{N_c} \int \frac{d\ell_T^2}{\ell_T^4} \int_{z_h}^1 \frac{dz}{z} \\ & \times D_{q \rightarrow h}(z_h/z) \frac{\alpha_s}{2\pi} C_A \left[\frac{1+z^2}{(1-z)_+} T_{qg}^A(x, x_L) \right. \\ & \left. + \delta(z-1) \Delta T_{qg}^A(x, \ell_T^2) \right], \end{aligned} \quad (6)$$

where

$$\begin{aligned} T_{qg}^A(x, x_L) = & \int \frac{dy^-}{2\pi} dy_1^- dy_2^- e^{i(x+x_L)p^+ y^- + ix_T p^+(y_1^- - y_2^-)} \\ & \frac{1}{2} \langle A | \bar{\psi}_q(0) \gamma^+ F_\sigma^+(y_2^-) F^{+\sigma}(y_1^-) \psi_q(y^-) | A \rangle \\ & \times (1 - e^{-ix_L p^+ y_2^-})(1 - e^{-ix_L p^+(y^- - y_1^-)}) \\ & \times \theta(-y_2^-) \theta(y_2^- - y_1^-), \end{aligned} \quad (7)$$

is quark-gluon correlation function which essentially contains four independent four-parton matrix elements in a nucleus and $x_T = \langle k_T^2 \rangle / 2p^+q^- = x_B \langle k_T^2 \rangle / Q^2$. With the definition of the $+$ functions [14], the term proportional to the δ -function accounts for virtual corrections and

$$\Delta T_{qg}^A(x, \ell_T^2) \equiv \int_0^1 dz \frac{1}{1-z} [2T_{qg}^A(x, x_L)|_{z=1} - (1+z^2)T_{qg}^A(x, x_L)]. \quad (8)$$

One can similarly get the contribution from the gluon fragmentation. We will neglect the radiative corrections to processes such as Fig. 1(b) that involve rescattering with a quark in the leading log approximation, because they can be shown to be proportional to $1/\ell_T^2$ [13] as compared to $1/\ell_T^4$ in Eq. (6).

Summing up all the leading contributions from LT and HT processes, we can effectively define the modified quark fragmentation function as

$$\frac{dW_{\mu\nu}}{dz_h} = \sum_q e_q^2 \int dx f_q^A(x, \mu_T^2) H_{\mu\nu}^{(0)}(x, p, q) \tilde{D}_{q \rightarrow h}(z_h, \mu^2). \quad (9)$$

where for completeness $f_q^A(x, \mu_T^2)$ should also include the HT contributions as studied by Mueller and Qiu [15]. The modified quark fragmentation function satisfies the following evolution equation

$$\frac{\partial \tilde{D}_q(z_h, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} [\tilde{\gamma}_{q \rightarrow qg}(z, x, x_L, \mu^2) \tilde{D}_q(z_h/z, \mu^2) + \tilde{\gamma}_{q \rightarrow qg}(1-z, x, x_L, \mu^2) D_g(z_h/z, \mu^2)], \quad (10)$$

with the modified splitting functions defined as

$$\tilde{\gamma}_{q \rightarrow qg}(z, x, x_L, \ell_T^2) = \gamma_{q \rightarrow qg}(z) + \Delta \gamma_{q \rightarrow qg}(z, x, x_L, \ell_T^2) \quad (11)$$

$$\Delta \gamma_{q \rightarrow qg}(z, x, x_L, \ell_T^2) = \frac{2\pi\alpha_s C_A}{\ell_T^2 N_c f_q^A(x, \mu_T^2)} \left[\frac{1+z^2}{(1-z)_+} T_{qg}^A(x, x_L) + \delta(1-z) \Delta T_{qg}^A(x, \mu^2) \right], \quad (12)$$

where $\gamma_{q \rightarrow qg}(z)$ is the normal splitting functions [10]. We assume in the leading order that the gluon fragmentation function follows the normal QCD evolution equations.

Solving the above equation is equivalent to summing all leading log twist-four contributions. As an approximation, one can write the solution as,

$$\begin{aligned} \tilde{D}_{q \rightarrow h}(z_h, \mu^2) &\equiv D_{q \rightarrow h}(z_h, \mu^2) + \Delta D_{q \rightarrow h}(z_h, \mu^2) \\ \Delta D_{q \rightarrow h}(z_h, \mu^2) &= \frac{\alpha_s}{2\pi} \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2} \int_{z_h}^1 \frac{dz}{z} \\ &\times [\Delta \gamma_{q \rightarrow qg}(z, x, x_L, \ell_T^2) D_{q \rightarrow h}(z_h/z, \mu^2) \\ &+ \Delta \gamma_{q \rightarrow gq}(z, x, x_L, \ell_T^2) D_{g \rightarrow h}(z_h/z, \mu^2)], \end{aligned} \quad (13)$$

where $D_{a \rightarrow h}(z_h, \mu^2)$ are the normal fragmentation functions. Notice that there is no collinear divergence in the above integration because of the LPM effect in T_{qg}^A . Because $\Delta \gamma$ is proportional to $1/\ell_T^2$, $\Delta D_{q \rightarrow h}$ is suppressed by $1/\mu^2$ relative to the LT fragmentation function $D_{q \rightarrow h}$.

To estimate the twist-four parton matrix elements, we generalize the approach by [9] to include the off-diagonal matrix elements. Assuming a Gaussian nuclear distribution in the rest frame, $\rho(r) \sim \exp(-r^2/2R_A^2)$, $R_A = 1.12A^{1/3}$ fm, we express T_{qg}^A in terms of single parton distributions,

$$T_{qg}^A(x, x_L) = \frac{C}{x_A} [f_q^A(x)(x_T + x_L)G(x_T + x_L) + f_q^A(x + x_L)x_T G(x_T)] (1 - e^{-x_L^2/x_A^2}), \quad (14)$$

where $G(x)$ is the gluon distribution per nucleon in a nucleus, $x_A = 1/MR_A$, and M is the nucleon's mass. The off-diagonal terms involves transferring momentum x_L between different nucleons inside a nucleus and thus should be suppressed for large nuclear size or large momentum fraction x_L . Notice that $\tau_f = 1/x_L p^+$ is the gluon's formation time. Thus, $x_L/x_A = L_A/\tau_f$ with $L_A = R_A M/p^+$ being the nuclear size in our chosen frame.

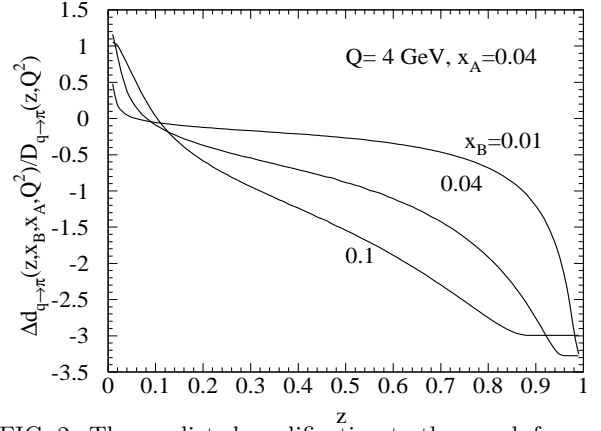


FIG. 2. The predicted modification to the quark fragmentation functions for three different values of initial quark energy $q^- = Q^2/2p^+ x_B$. $x_A = 0.04$ corresponds to $A \approx 200$

Using the above approximation in Eq. (13) and replacing the momentum fraction x_L in the parton distributions by its average value $\langle x_L \rangle \sim x_A$, we have

$$\begin{aligned} \Delta D_{q \rightarrow h}(z_h, Q^2) &\approx \frac{C_A \alpha_s^2}{N_c} \frac{\tilde{C}}{Q^2 x_A^2} \Delta d_{q \rightarrow h}(z_h, x_B, x_A, Q^2), \\ \tilde{C} &= C[x_T G(x_T) + (x_T + x_A)G(x_T + x_A)], \end{aligned} \quad (15)$$

where we choose the factorization scale $\mu^2 = Q^2$. Shown in Fig. 2 are the numerical results of $\Delta d(z_h, x_B, x_A, Q^2)$ for three different values of x_B . The parameterization in Ref. [16] of the normal fragmentation functions is used in our calculation. As we see from the numerical results, the modification to the shape of fragmentation function increases for larger values of x_B corresponding to smaller quark energy q^- at fixed Q^2 . The modification also increases for smaller values of x_A corresponding to larger

nuclear size. As shown in Eq. (15), the magnitude of the modification depends quadratically on the nuclear size. This is because the LPM effect modifies the transverse momentum spectra of the radiated gluon such that the phase space for the momentum integral is limited. That limited phase space is proportional to the nuclear size. Together with the linear dependence of the parton correlation function, this leads to non-linear dependence of the total energy loss on the nuclear size.

We can define the quark energy loss as the momentum carried by the emitted gluons. From Eq. (13), we have,

$$\begin{aligned} \langle \Delta z_g \rangle &= \int_0^{Q^2} \frac{d\ell_T^2}{\ell_T^2} \int_0^1 dz \frac{\alpha_s}{2\pi} z \Delta\gamma_{q \rightarrow gq}(z, x, x_L, \ell_T^2) \\ &\approx \frac{C_A \alpha_s^2}{N_c} \frac{\tilde{C}}{Q^2} \frac{x_B}{x_A^2} 6 \ln\left(\frac{1}{2x_B}\right), \quad (x_A \ll x_B \ll 1). \end{aligned} \quad (16)$$

According to Ref. [17,18], \tilde{C} can be related to the transverse momentum broadening of Drell-Yan dilepton in pA collisions,

$$\Delta \langle k_T^2 \rangle_{DY} \approx \frac{2\pi\alpha_s}{N_c} \frac{\tilde{C}}{2x_A} \approx 0.022 A^{1/3} \text{ GeV}^2. \quad (17)$$

In the rest frame of the nucleus, the total energy loss is

$$\Delta E = q_0 \langle \Delta z_g \rangle \approx 0.35 \alpha_s A^{2/3} \ln \frac{1}{2x_B} \text{ GeV}, \quad (18)$$

which depends quadratically on the nuclear size. For $x_B = 0.1$, $\alpha_s = 0.3$ and $A = 200$, $\Delta E \approx 5.8 \text{ GeV}$ which is consistent with the estimate in Ref [4].

In summary, we have derived for the first time the evolution equations for the nuclear modified quark fragmentation functions due to multiple scattering and induced radiation. We can generalize our results to the case of hot QCD matter. In our approximation, Eq. (14), the quark distribution which represents the initial quark production probability does not enter into the modified evolution equations. What remains is the gluon matrix element. In the case of a hot QCD medium, the coordinates of the two gluon fields are not restricted to the nucleon size due to deconfinement in a nucleus. Rather, the final results will be sensitive to the correlation length of the hot medium [19]. If there is a dramatic change of the correlation length during the QCD phase transition, one should also see a big change in the modification of the parton fragmentation functions.

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